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THE MOVEMENT OF WATER DROPLETS IN CLOUDS AROUND THE NOSE OF AN ATMOSPHERIC RESEARCH AIRCRAFT

P. Feuillebois, M. F. Scibilia

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16. Abstract

This report describes the procedure for calculating concentrations of water droplets in clouds around the nose of an aircraft. An attempt is made to evaluate the dynamic interactions between droplets and air flows. It is demonstrated that the interactions between droplets in a cloud are negligeable (each particle behaves as if it were the only one in the air). It is also shown that the air flow carrying the droplets is not influenced by the presence of these droplets. These 2 results make it possible to study the trajectory of each droplet separately after calculating the air flow on the assumption that it is dry. The Langragian method is used to calculate the concentrations, which are calculated along two close trajectories.

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THE MOVEMENT OF WATER DROPLETS IN CLOUDS
AROUND THE NOSE OF AN ATMOSPHERIC RESEARCH AIRCRAFT

P. Feuillebois, M. Scibilia
Polyphasic Flow Team
CNRS* Aerothermal Laboratory

INTRODUCTION /1**

The underlying technical problem presented by this study is the movement of water droplets in clouds around the nose of an aircraft. In practice, this consists of collecting samples of suspended droplets which are representative of the cloud, for the purpose of future meteorological investigations. It is natural to think that larger drops will break over it. The proportion of the different droplet sizes, and their respective concentrations would therefore be different on each point around the aircraft nose. The position of the droplet sensor tube, in front of the aircraft nose, should therefore influence the samples collected. In this report, we will calculate the droplet concentrations, to determine more precisely what they are.

From the theoretical standpoint, we tried to evaluate the dynamic interactions between droplets and air flows. We will show later in this report that the interactions between droplets in a cloud are negligeable. In other words, each particle behaves (for the calculation of the forces exerted upon it) as if it were the only one in the air. Moreover, we will show that, in most cases, the flow of air carrying droplets is not influenced on the average by the presence of these droplets. These two results then make it possible to study separately the trajectory of each droplet after having calculated the flow of air assumed to be dry. To calculate the concentrations, we will introduce the Langragian method. The concentration is calculated directly according to two close trajectories.

*National Center for Scientific Research.
**Numbers in the margin indicate pagination in the original text.

Experiments are performed at the National Astronomy and Geophysics Institute [I.N.A.G.] on an atmospheric research aircraft. The technical data were reported to us by GRIBKOFF, in charge of devising these experiments.

The first position of the sensor tube is schematized in figure 1. The numerical data are the following:

-aircraft speed: $v_\infty = 60$ m/s,

-pressure: $p = 800$ to 1100 mb,

-temperature: $T_\infty = -10°C$ to $+30°C$ /2

-saturating steam concentration: 10 to 15 g/m^3 of air

-water droplets:

  .diameter: $d = 1$ to 50 μm

  .concentration: $m = 1$ g/m^3 of air on the average

  3 to 4 g/m^3 of air maximum

  .number of droplets: $n = 200$ to 500 per cm^3 of air.

1 - CALCULATION OF DIMENSIONLESS NUMBERS AND DISCUSSION OF THE PHYSICAL CONDITIONS /3

1.1. REYNOLDS NUMBER

To determine the rate of flow about the nose of an aircraft (assumed to be hemispherical, of radius 0.7 m), let us calculate the Reynolds number.

To accomplish this, the air viscosity must be evaluated.

Dry Air: According to SCHLICHTING [1] we have:

T = (°C)	-10	0	10	20	30
ν = (m²/s)	1.208×10^{-5}	1.301×10^{-5}	1.394×10^{-5}	1.486×10^{-5}	1.6×10^{-5}

Influence of Humidity:

According to [2], the saturating water steam pressure p_s is:

T = (°C)	0	10	20	30
p_s (bars)	$6{,}23 \times 10^{-3}$	1.252×10^{-2}	$2{,}381 \times 10^{-2}$	$4{,}311 \times 10^{-2}$

The molar concentration is maximum for 30°C and therefore:

$$X = \frac{p_s}{p} \simeq 4.3x.0^{-2}$$

Such a small molar concentration does not influence the viscosity [2].

Consistency of data pertaining to humidity: /4

At 10°C, we have a number of air moles per m^3 of air:

$$n_{air} = \frac{1000}{22.4} \times \frac{273}{283} = 43$$

This gives a water weight per m^3 of air:

$$m_{H_2O} = 18\ n_{H_2O} = 18 \times n_{air} = 18 \times 1.252 \times 10^{-2} \times 43 = 9.1\ g$$

which is consistent with the datum 10 to 15 g of saturating steam per m^3 of air.

Influence of Water Droplets:

According to the data, m = 1 g of water/m^3 of air on the average. The concentration of droplets per unit of volume is therefore, with $\rho_p = 10^6\ g/m^3$ (water density):

$$c = \frac{\bar{m}}{10^6\ g/m^3} = 10^{-6} .$$

The viscosity of a diluted suspension is given in a first approximation by Einstein's formula, on the assumption that the water droplets follow the air flow:

$$\mu = \mu_{air}\ (1 + \frac{5}{2}).$$

Since $c = 10^{-6}$, we see that the variation of μ is negligeable. In regard to the density of the mixture, the concentration of droplets is about:

$$\frac{m}{\rho_{air}} = \frac{1g}{1300\ g} \approx 1\%,$$ and is therefore negligeable also.

Finally, the water droplets do not influence the kinematic viscosity $\nu = \frac{\mu}{\rho}$.

In conclusion, the <u>kinematic viscosity</u> of humid air loaded with water droplets in clouds is virtually equal to that of dry air.

The <u>Reynolds number</u> based on the speed, v_∞ of the aircraft and on the radius R, of the hemispheric nose of the aircraft has the following values:

T = (°C)	-10	0	10	20	30
$Re = \frac{v_\infty R}{\nu} =$	3.48×10^6	3.23×10^6	$3,01\times10^6$	2.83×10^6	$2,63\times10^6$

According to SCHLICHTING, the flow about a sphere placed in a flow with little turbulence becomes turbulent for:

Re is greater than or equal to 1.92×10^5

Moreover, the atmosphere, where the turbulence is characterized by large vortices, may be considered on the sphere scale as having little turbulence. /5

The turbulent flow about a sphere is characterized by the boundary layer separation situated downstream from the midship bulkhead. The flow is turbulent in this region. However, downstream, the flow is virtually that of an inviscid flow.

This is the case here where

Re is greater than or equal to 2.6×10^6

The turbulent region thrown downstream has no influence on the flow about the hemisphere, which is an inviscid flow.

1.2. POTENTIAL FLOW ABOUT THE SPHERE

The inviscid fluid flow about the sphere is a potential flow, which may be expressed in cylindrical coordinates (r,z) (figure 1).

$$\phi = - v_\infty z - \frac{v_\infty z}{2} \frac{R^3}{(r^2 + z^2)^{3/2}}$$

The components of the relative air speed relative to the aircraft are, according to r,z, respectively:

$$v_r = \frac{\partial \phi}{\partial r} = \frac{3}{2} v_\infty z r \frac{R^3}{(r^2 + z^2)^{5/2}}$$

$$v_z = \frac{\partial \phi}{\partial z} = - v_\infty - \frac{v_\infty}{2} R^3 \frac{(r^2 - 2z^2)}{(r^2 + z^2)^{5/2}}$$

For the sake of convenience, we will employ dimensionless magnitudes using R as the reference length, and v_∞ as the reference speed. Let us put:

$$\rho = \frac{r}{R}$$

$$\zeta = \frac{z}{R}$$

$$V_\rho = \frac{v_r}{v_\infty} = \frac{3}{2} \frac{\zeta\rho}{(\zeta^2 + \rho^2)^{5/2}}$$

$$V_\zeta = \frac{v_z}{v_\infty} = -1 - \frac{1}{2} \frac{(\rho^2 - 2\zeta^2)}{(\zeta^2 + \rho^2)^{5/2}}$$

At the measuring point shown in figure 1 (tube end), the reduced cylindrical coordinates are: /6

$$\rho_M = \frac{r_M}{R} = 0{,}571 \ ,$$

$$\zeta_M = \frac{\sqrt{R^2 - r_M{}^2} + \ell_M}{R} = 1{,}18 \ .$$

On this point, the reduced speed components calculated are equal to:

$$|V_\rho|_M = 0.2626$$

$$|V_\xi|_M = -0.6817$$

The angle which the speed forms with the z (or ξ) axis is:

$$\theta = \text{Arctg} \left|\frac{V\rho}{V_\xi}\right| = 21°$$

and the reduced speed modulus is:

$$V = \sqrt{V_\rho{}^2 + V_\zeta{}^2} = 0{,}7305 \ .$$

We see that the measuring point is within a region where the speed is fairly inclined over the axis, and where the modulus is appreciably smaller than the value at infinity.

As nothing leads us to suppose that the droplets follow the fluid flow, the movement of the droplets should be studied in more detail.

1.3. STOKES NUMBER

This number characterizes the movement of particles with respect to the movement of air. It is expressed:

$$SK = \frac{\tau_p}{\tau_e}$$

$-\tau_p$ is the deceleration time of a droplet. On the assumption that the droplet is subjected to the Stokes drag, this time is about:

$$\tau_p \doteq \frac{\frac{4}{3}\pi a^3 \rho_p}{6 \pi a \mu}$$ (at the radius of the droplet):

$-\tau$ is the characteristic time of the air flow about the nose of an aircraft:

$$\tau_e = \frac{R}{V_\infty} .$$

We therefore calculate: /7

$$SK = \frac{2}{9} \text{Re} \ \frac{\rho_p}{\rho} \left(\frac{a}{R}\right)^2$$

With the values calculated for Re, and the values for air density (assuming an inviscid gas):

$$\rho = 1.29 \frac{273}{T(k)} \frac{p(mb)}{1013} \ (kg/m^3)$$

we obtain the following values of $\frac{2}{9}$ Re $\frac{\rho_p}{\rho}$:

Values of $\frac{2}{9}$ Re $\frac{\rho_p}{\rho}$: (to be multiplied by 10^8):

T (°C) \ p (mb)	800	1013	1100
-10	7.30	3,77	5,33
0	7,04	5,56	5,13
10	6 93	5.39	4,95
20	6,62	5,24	4.80
30	6.35	5,04	4,64

Consistency of the Data With the Droplet Sizes Under Consideration

Above, we calculated that the droplet density concentration is on the average $c = 10^{-6}$ for $m = 1$ g water/m^3. At the maximal value $m = 3$ to 4 g water/m^3 of air therefore $c = 3$ to 4×10^{-6}. Another datum is the number of droplets

$$n = 200 \text{ to } 500/cm^3$$

On the basis of c and n we may deduce the droplet diameter (already given; the data are superabundant):

$$d = \left(\frac{6}{\pi} \frac{c}{n}\right)^{1/3}$$

We calculate the values of d:

c / (n/cm^3)	10^{-6}	$4x10^{-6}$
200	21	33
500	15	25

the droplet diameter is within the interval 15-30 µm (consistent with the data d = 1 to 50 µm). The most common interval found is 15-30 µm.

Given the preceding, let us calculate the typical values ofthe Stokes number:

d = 2a (µm)	SK		
	1100 mb 30°C	1013 mb 0°C	800 mb -10°C
1	$2,37x10^{-4}$	$2,84x10^{-4}$	$3,72x10^{-4}$
5	$5,92x10^{-3}$	$7,09x10^{-3}$	$9,31x10^{-3}$
10	$2,37x10^{-2}$	$2,84x10^{-2}$	$3,72x10^{-2}$
20	$9,47x10^{-2}$	$1,13x10^{-1}$	$1,49x10^{-1}$
30	$2,13x10^{-1}$	$2,55x10^{-1}$	$3,35x10^{-1}$
40	$3,79x10^{-1}$	$4,54x10^{-1}$	$5,96x10^{-1}$
50	$5,92x10^{-1}$	$7,09x10^{-1}$	$9,31x10^{-1}$
52	-	-	1,01
60	-	1,02	-
65	1,00	-	-

Physically, a low Stokes number corresponds to droplets which have a tendency to easily follow the air movement. This is the case of droplets whose size is smaller than or equal to 10 µm. A high Stokes number corresponds to droplets which continue a straight forward movement, and are not at all influenced by the movement of air. For the majority of the droplets considered here, a Stokes number of order 1 corresponds to an

intermediary state in which the droplets follow the fluid flow "slightly". This is theoretically the most complicated case where it is necessary to calculate the trajectory of each type of droplet individually.

2 - EQUATIONS FOR THE SUSPENSION OF OF DROPLETS IN ITS MOVEMENT AND NUMERICAL CALCULATIONS /9

2.1. DRAG FORCE ON THE DROPLETS

In the expression of the Stokes number, we have assumed that one droplet is subjected to the Stokes drag force. This formula is valid for droplets that are fairly small such that the Reynolds number of the relative flow about each droplet remains low. We will see below that this condition remains verified, except for the largest particles of 50 μm for which we shall give a correction for the drag.

A second condition for applying the Stokes formula is that the droplets behave dynamically as solid spheres, i.e. that the movement of liquid in the droplet may be disregarded. According to LEVICH [4], this is verified for diameters between 0.5 μm and 10 μm. Without more date, we will still keep this assumption for larger diameters.

The Stokes force is valid for an isolated particle. However for a suspended particle, the interactions between particles may be involved. However the correction to be made is of the order of c, and since $c \simeq 10^{-6}$, it is discardable here.

The fact that c is small implies that the droplets behave as specific forces (their volume is negligeable).

2.2. EQUATIONS GOVERNING SUSPENSION

With the preceding assumptions, we will express for the

mixture of fluid flow and droplets a system of movement equations, according to MARBLE. These equations are generalized here to N types of droplets; they are expressed as dimensionless (see Feüillebois' thesis):

-Fluid continuity: $\vec{\nabla} . \vec{V} = 0$

-Fluid momentum:

$$(\vec{V} . \vec{\nabla})\vec{V} = - \vec{\nabla}P + \frac{1}{R_e} .\nabla^2\vec{V} + \sum_{i=1}^{N} f_i \frac{(\vec{V}_{pi} - \vec{V})}{Sk_i}$$

-Continuity of type i particles: $\vec{\nabla} . (f_i \vec{V}_{pi}) = 0$

-Momentum of type i particles:

$$(\vec{V}_{pi} . \vec{\nabla})\vec{V}_{pi} = (\vec{V} - \vec{V}_{pi})/SK_i$$

with the following dimensionless variable: $\vec{V}$ fluid velocity, P fluid pressure, and , for type i particles, V_{pi}, velocity, f_i, concentration per unit of mass (mass of particles per unit of fluid mass), Sk_i Stokes number.

It is interesting to observe that the term $f_i(\vec{V}_{pi}-\vec{V}/SK_i$ for the effect of the particles on the flow is negligeable. Actually, if we calculate the concentration per unit of mass f_i from the numerical data, we find: $f_i = m_i/\rho$ where $m_i = 1$ to 4 g/m^3 of air, $\rho = 0.92$ to 1.45 kg/m^3. Therefore, f_i is less than or equal to 4.3×10^{-3}.

The term $f_i(\vec{V}_{pi} - \vec{V})/Sk_i$ could be nondiscardable if SK_i were small at the same time as f_i, i.e. if the small particles were the most abundant in the mass. However, we have seen earlier that the most abundant particles are in the range 15-30 μm.

The interaction term $f_i(\vec{V}_{pi}-\vec{V})/Sk_i$ is therefore always discardable.

Consequently, the air flow about the aircraft nose is not modified by the presence of particles. The system of equations cross-checks and it is simply necessary to solve independently the system pertaining to each type of particle. The equation for the momentum of the particles is also expressed in Lagrangain form, and in cylindrical coordinates:

$$\frac{dV_{p\rho}}{dT} = \frac{V_\rho - V_{p\rho}}{Sk}$$

$$\frac{dV_{p\zeta}}{dT} = \frac{V_\zeta - V_{p\zeta}}{Sk}$$

d/dT is the derivative with respect to time calculated on the basis of one particle in its movement.

T is the reduced time /11

$$T = \frac{t\ v_\infty}{R}$$

Furthermore, and to simplify the annotations, we have eliminated index i relative to each type of particle. To calculate the droplet trajectory, the values given above of V_ρ, V_ζ for the inviscid fluid flow and the expressions of the particle velocity:

$$V_{p_\rho} = \frac{d\rho}{dT}$$

$$V_{p_\zeta} = \frac{d\zeta}{dT}$$

If we combine these equations, we obtain the following system to solve:

$$\frac{dV_{p_\rho}}{dT} = \left(\frac{3}{2}\ \frac{\eta\,\zeta}{(\eta^2 + \zeta^2)^{5/2}}\right)\frac{1}{Sk} \quad , \quad \frac{d\rho}{dt} = V_{p\rho}$$

$$\frac{dV_\zeta}{dT} = \left(-1 - \frac{1}{2}\ \frac{\rho^2 + 2\zeta^2}{(\rho^2 + \zeta^2)^{5/2}} - V_{p\zeta}\right)\frac{1}{Sk} \quad , \quad \frac{d\zeta}{dT} = V_{p\zeta}$$

The initial conditions to be used express that at upstream infinity the droplets have the velocity of air:

$$T \to -\infty \ : \quad \rho \to \rho_\infty \ ; \ \zeta \to \infty$$
$$V_{p_\rho} \to 0 \ ; \ V_{p_\zeta} \to -1 \ ;$$

For the numerical calculation, we replaced this condition at infinity by the following condition:

$$T = 0: \quad \rho = \rho_o ; \ \zeta \to \infty$$

$$V_{p_\rho} \to 0; \ V_{p_\zeta} \to -1;$$

which proved to give the required accuracy.

2.3. CALCULATIONS OF THE TRAJECTORIES

The integration of the differential system was achieved using the DREBS sub-program available at CIRCE. The sub-program, in FORTRAN, performes dual-precision calculations.

The droplets trajectories were calculated, then sketched for four cases:

Sk = 0.01; 0.1; 0.5; 1 /12

characteristic of the droplet sizes studied.

In figures 2 to 5, we see that the smallest droplets (Sk = 0.01 by-pass the aircraft nose, whereas the largest (Sk = 0.5 Sk = 1) will hit against the aircraft nose, as could be expected, according to the definition of the Stokes number (see above).

It is also remarkable to note that these numerical results coincide with the theoretical calculation of MICHAEL [5] who predicts that the particles collide with the sphere for Sk is greater than 0.12.

The trajectories were calculated in detail near the opening of the measuring probe, or near:

$$\rho_M = 0.571 \pm 0.011$$
$$\zeta_M = 1.18$$

2.4 CALCULATION OF THE PARTICLES CONCENTRATION

We replace the equation for the particles continuity in a form more adaptable to the present numerical calculation.

For each type of particle (fixed Sk), let us consider two very close trajectories, each differing from the other by the initial value of ρ (figure 6).

$$\rho = \rho$$
$$\rho = \rho_o + \Delta\rho_o$$

We express that the total particles flux across the axisymetric surface made up of two surfaces of particles streams, and of end crowns, is zero:

We have:

$$f\,\vec{V}_p \cdot \vec{\Delta S} + f_o\,\vec{V}_{p_o} \cdot \vec{\Delta S}_. = 0$$

Using a little geometry (figure 6), gives us:

$$|\vec{\Delta S}| = 2\pi\rho\ \Delta h$$

$$\Delta h = \delta\rho\ \cos\alpha$$

$$\delta\rho = \Delta\rho + \Delta\zeta\ \sin\alpha$$

where: $$\sin\alpha = \frac{V_{p_\rho}}{\sqrt{V_{p_\rho}^2 + V_{p_\zeta}^2}}$$ /13

therefore:

$$|\vec{\Delta S}| = 2\pi\rho\ \ (\Delta\rho + \Delta\zeta\ \sin\alpha)\ \cos\alpha$$

$$\vec{V}_p \cdot \vec{\Delta S} = 2\pi\rho\ (\Delta\rho + \Delta_\zeta\ \sin\alpha)\ \cos\alpha\ V_p$$

$$= -2\pi\rho(\Delta\rho + \Delta\zeta\ \sin\alpha)\ V_{p\zeta}$$

$$\vec{V}_{p_o} \cdot \vec{\Delta S}_o = -2\pi\rho_o\ \Delta\rho_o$$

The conservation of flux is therefore expressed:

$$F = \frac{f}{f_o} = -\frac{\rho_o}{\rho}\ \frac{V_p\ \Delta\rho_o}{(V_p\ \Delta\rho + V_{p_\rho}\ \Delta\zeta)V_{p_\zeta}}$$

where: $$V_p = \sqrt{V_{p_\rho}^2 + V_{p_\zeta}^2}$$

We can therefore calculate on the basis of two close projecteries, the ratio of the concentration at the measuring point f to the concentration at infinity f_o.

The error with such a method may be considerable if the trajectories are not evaluated with sufficient accuracy, as the study below will show.

2.5. CALCULATING THE ERROR ON THE VALUE OF CONCENTRATION F

$$F = -\frac{\rho_o}{\rho} \frac{v_p \Delta\rho_o}{(v_p \Delta\rho + v_{p_\rho} \Delta\zeta) v_{p_\zeta}}$$

We will note δX the error range on the quantity X. The relative error on F is calculated:

$$\frac{\delta F}{F} = \frac{\delta\rho_o}{\rho_o} + \frac{\delta\rho}{\rho} + \frac{\delta v_p}{v_p} + \frac{\delta(\Delta\rho_o)}{\Delta\rho_o} + \frac{\delta v_{p_\zeta}}{|v_{p_\zeta}|} + \frac{\delta[v_p \Delta\rho + v_{p_\zeta} \Delta\zeta]}{v_p \Delta\rho + v_{p_\rho} \Delta\zeta}$$

ρ_o is the initial value set for ρ, therefore $\delta\rho_o = 0$;
$\rho_o + \Delta\rho$ is another intial value, therefore $\delta(\Delta\rho) = 0$

$$v_p = \sqrt{v_{p_\rho}^2 + v_{p_\zeta}^2}$$

$$\delta v_p \doteq \frac{|v_{p_\rho}| \, \delta v_{p_\rho} + |v_{p_\zeta}| \delta v_{p_\zeta}}{v_p}$$

$$\delta[v_p \Delta\rho + v_{p_\rho} \Delta\zeta] = v_p \, \delta(\Delta\rho) + \Delta\rho \, \delta v_p + v_{p_\zeta} |\delta(\Delta\zeta) + |\Delta\zeta| \, \delta v_{p_\rho}$$ /15

Let us take the example of calculations made for Sk = 1, the tolerance being 10^{-5}. We obtained the following results:

$\rho_o = 0,52$	$T = 8,9$: $\rho = 0,56510$	$\zeta = 1,2310$	$V_{P_\rho} = 0,074882$	$V_{P_\zeta} = -0$
	$T = 9,0$: $0,57342$	$1,1450$	$0,092198$	-0
$\rho_o = 0,53$	$T = 8,9$: $0,57568$	$1,2302$	$0,075503$	-0
	$T = 9,0$: $0,58406$	$1,1441$	$0,092837$	-0
$\rho_o = 0,54$	$T = 8,9$: $0,58624$	$1,2294$	$0,076091$	-0
	$T = 9,0$: $0,59468$	$1,1431$	$0,093432$	$-0,$

Checking the calculation accuracy on a computer:

For

$$\left.\begin{array}{l} \rho|_{\rho_o = 0.54} - \rho|_{\rho_o = 0,53} = 0,01056 \\ \rho|_{\rho_o = 0,59} - \rho|_{\rho_o = 0,52} = 0,01058 \end{array}\right\} \text{précision } 10^{-5}$$

$T = 9,0$:

$$\left.\begin{array}{l} \rho|_{\rho_o = 0.54} - \rho|_{\rho_o = 0,53} = 0,01062 \\ \rho|_{\rho_o = 0.53} - \rho|_{\rho_o = 0,52} = 0,01064 \end{array}\right\} \text{précision } 10^{-5}$$

$T = 8,9$

$$\left.\begin{array}{l} \zeta|_{\rho_o = 0,54} - \zeta|_{\rho_o = 0,53} = -0,00080 \\ \zeta|_{\rho_o = 0,53} - \zeta|_{\rho_o = 0,52} = -0,00080 \end{array}\right\} \text{précision } 10^{-4}$$

$T = 9,0$

$$\left.\begin{array}{l} \zeta|_{\rho_o = 0.54} - \zeta|_{\rho_o = 0,53} = -0,00100 \\ \zeta|_{\rho_o = 0,53} - \zeta|_{\rho_o = 0,52} = -0,00090 \\ \zeta|_{\rho_o = 0.55} - \zeta|_{\rho_o = 0,54} = -0,00100 \\ \zeta|_{\rho_o = 0,52} - \zeta|_{\rho_o = 0,51} = -0,00100 \end{array}\right\} \text{précision } 10^{-4}$$

We evaluate:

$$\delta\rho = 10^{-5} \; ; \; \delta\zeta = 10^{-4} \; ;$$

$$\delta(\Delta\rho) = 10^{-5} \; ; \; \delta(\Delta\zeta) = 10^{-4} \; ;$$

and

$$; \; \delta V_{P_\rho} = 10^{-5} \; ; \; \delta V_{P_\zeta} = 10^{-5}$$

If we consider the values:

$$\rho_o = 0,53 \; ; \; T = 8.9 \; ; \; \rho = 0,56510 \; ; \; \zeta = 1,2310 \; ; \; V_{P_\rho} = 0.074882 \; ; \; V_{P_\zeta} = 0,86767$$

$\Delta\rho_o = 0{,}01$; $\Delta\rho = 0{,}01058$; $\Delta\zeta = -0{,}00080.$

we obtain:
$$\frac{\delta F}{F} = 1{,}49.10^{-4}$$

The most important contribution comes from the error in $\Delta\zeta$, due to the error in $[V_p\ \Delta\rho + V_{p_\rho}\ \Delta\zeta]$

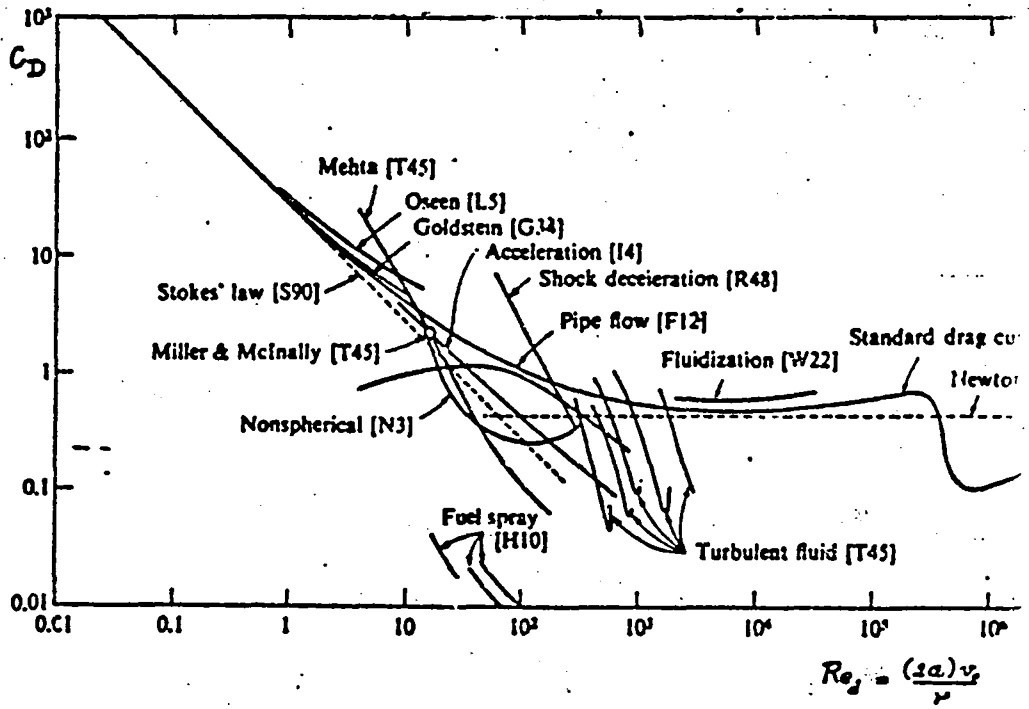
2.6. VALUES CALCULATED FOR CONCENTRATION

Sk	T	Tube de courant ρ_o	$\rho_o + \Delta\rho_o$	$F = f/f_\infty$	
0,01	9,1	0,41	0,42	1,00126	$1{,}000 \pm 3x10^{-3}$
		0,42	0,43	1,00465	
		0,43	0,44	0,99965	
0,1	9,1	0,43	0,44	1,02499	$1{,}022 \pm 3x10^{-3}$
		0,44	0,45	1,02175	
		0,45	0,46	1,02165	
0,5	9,1	0,48	0,49	1,0348	
		0,49	0,50	1,03089	$1{,}029 \pm 2x10^{-3}$
		0,50	0,51	1,02719	
1,0	9	0,51	0,52	1,0126	
		0,52	0,53	1,0116	$1{,}012 \pm 10^{-3}$
		0,53	0,54	1,0127	
0,3	9	0,48	0,49	1,0229	$1{,}023 \pm 3x10^{-3}$
		0,49	0,50	1,0205	
0,7	9	0,48	0,49	1,017	
		0,49	0,50	1,015	$1{,}015\ \ 2x10^{-3}$

The results are given in figures 7 and 8.

2.7. CORRECTIONS ON THE DRAG FOR LARGE PARTICLES

When the Reynolds number of the relative flow with respect to one particle is low, we may use Stoke's force. This is no longer true if the Reyonlds number, based on the diameter, becomes greater than 1. (see the figure extracted SOO's book [6].



The relative Reynolds number based on the diameter

$$Re_d = \frac{2a|v_p - v|}{\nu}$$

may be evaluated according to the results already calculated using Stoke's force on each droplet. The number Re_d will increase with the size of the particles, for which the error will greater.

Let us take the example: $2a = 50\ \mu m$ (for these droplets, $Sk \simeq 1$).

According to the calculation results, the velocities at the measuring point are:

$$v_p = 0{,}2626 \; ; \; v_\zeta = -0{,}6817$$

$$v_{p_\rho} = 0{,}07489 \; ; \; v_{p_\zeta} = -0{,}86765$$

Therefore

$$|v_p - v| = \sqrt{(v_{p_\rho} - v_\rho)^2 + (v_{p_\zeta} - v_\zeta)^2}$$

and

$$Re_v = \frac{2a\, v_\infty\, |v_p - v|}{\nu} = \frac{50\text{x}10^{-6} \text{ x } 50\, |v_p - v|}{1.5\text{x}10^{-5}} = 52{,}84$$

For this Reynolds number, the coefficient of the sphere's drag (see "Standard drag curve") is, according to the figure, about 2.45 times that of Stokes law. There is no good empirical formula giving the coefficient of drag for intermediary Reynolds numbers. The least undesirable one still seems to be that of KLYACHKO [see FUCHS, 1966 [7]) and PUTMAN (1961), cited by RUDINGER [8]. /17

$$C_D = \frac{24}{Re_d} \left(1 + \frac{1}{6} Re_d^{2/3}\right)$$

where Stoke's coefficient, $\frac{24}{R_{e_d}}$ is multiplied by an empirical factor. For $R_{e_d} = 52.84$, this factor is equal to 3.35. The curve borrowed by SOO gave the factor 2.45, but we should rather have more confidence in the formula above. RUDINGER actually indicated, through several comparisons with experience, that its accuracy is within a few %.

To convert the formulas, we simply have to multiply Stoke's coefficient $6\pi a\mu$ by $(1 + \frac{1}{6} Re_d{}^{2/3})$ which is the same as dividing the Stokes number

$$SK = \frac{4/3\ \pi\ ^3\ \rho_p}{6\pi a\mu} \text{ by } \left(1 + \frac{1}{6}\ Re_d{}^{2/3}\right)$$

In this factor Re_d is variable:

$$Re_d = \frac{2a \; v_\infty}{\nu} \; |V_o - V|$$

We introduce the new dimensionless numbers according to Reynolds number $Re = RV_\infty/\nu$ of the aircraft and the adimensional size of droplet $D = \frac{2a}{R}$. Therefore:

$$Re_d = D \; Re \; |V_p - V|$$

The system to be integrated is therefore expressed:

$$\frac{dV_{p_\rho}}{dT} = (V_\rho - V_{p_\rho}) \; (1 + \frac{1}{6} [D \; Re \; (V_p - V)]^{2/3}) \; \frac{1}{Sk}$$

$$\frac{dV_{p_\zeta}}{DT} = (V_\zeta - V_{p_\zeta}) \; (1 + \frac{1}{6} [D \; Re \; (V_p - V)]^{2/3}) \; \frac{1}{Sk}$$

with:

$$V_\rho = \frac{3}{2} \; \frac{\rho \, \zeta}{(\rho^2 + \zeta^2)^{5/2}}$$ /18

$$V_\zeta = -1 - \frac{1}{2} \; \frac{(\rho^2 - 2\zeta^2)}{(\rho^4 + \zeta^2)^{5/2}}$$

$$V = \sqrt{V_\rho^2 + V_\zeta^2}$$

$$V_p = \sqrt{V_{p_\rho}^{\;2} + V_{p_\zeta}^{\;2}}$$

and

$$\frac{d\rho}{dT} = V_{p_\rho}$$

$$\frac{d\zeta}{dT} = V_{p_\zeta}$$

D and R_e values:

The diameter 2a of the droplets varies from 1 to 60 μm. Therefore:

$$D = \frac{2a}{R} + \frac{10^{-6}}{0.7} \text{ to } \frac{60x10^{-6}}{0.7} = 1.43x10^{-6} \text{ to } 8.571x10^{-5}$$

Re varies from $3.48x10^6$ (at -10°C) to $2.6x10^6$ (at 30°C). Therefore $3.76 < D\ Re < 298$

Values Selected For the Calculation:

We will select the mean values corresponding to:

$$T = 0°C; \quad p = 1013 \text{ mb}$$

Therefore

$$Re = 3.23x10^6$$

$$R_p = 7.675x10^2 \quad \text{we set } R_p = \frac{\rho_p}{\rho}$$

$$Sk = \frac{2}{9} Re \frac{\rho_p}{\rho} \left(\frac{a}{R}\right)^2 = \frac{1}{18} Re\ R_p\ D^2$$

by setting $D = \frac{2a}{R}$.

Therefore: $D = \sqrt{\frac{18\,Sk}{Re\ R_p}}$

We select for Sk the values 10^{-2}; 10^{-1}; 0.5; 1.

At the temperature and pressure selected, these values correspond to the droplets diameters:

Sk	D	d = DR
0.01	$8.52x10^{-6}$	5.94 μm
0.1	$2.68x10^{-5}$	18.8 μm
0.5	$6.00x10^{-5}$	42.0 μm
1	$8.52x10^{-5}$	59.4 μm

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3 CONCLUSION FOR THE AIRCRAFT WITH HEMISPHERIC NOSE /20

Even with the drag correction, the values of the concentration F remain close to one. This means that, to the nearest 3%, the values f measured in the tube will actually be the values f_o in the cloud.

The 3% deviation is the maximum, corresponding to droplets of about 40 μm. For smaller or larger droplets, this deviation is less.

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FIGURES

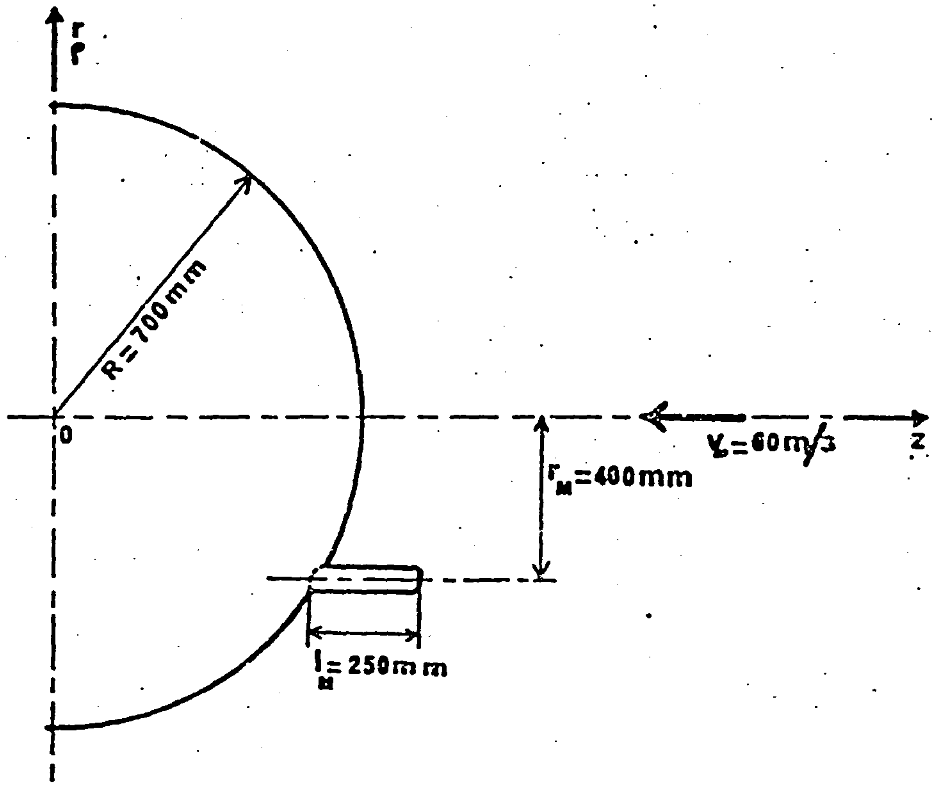


Fig 1. Diagram of the aircraft nose.

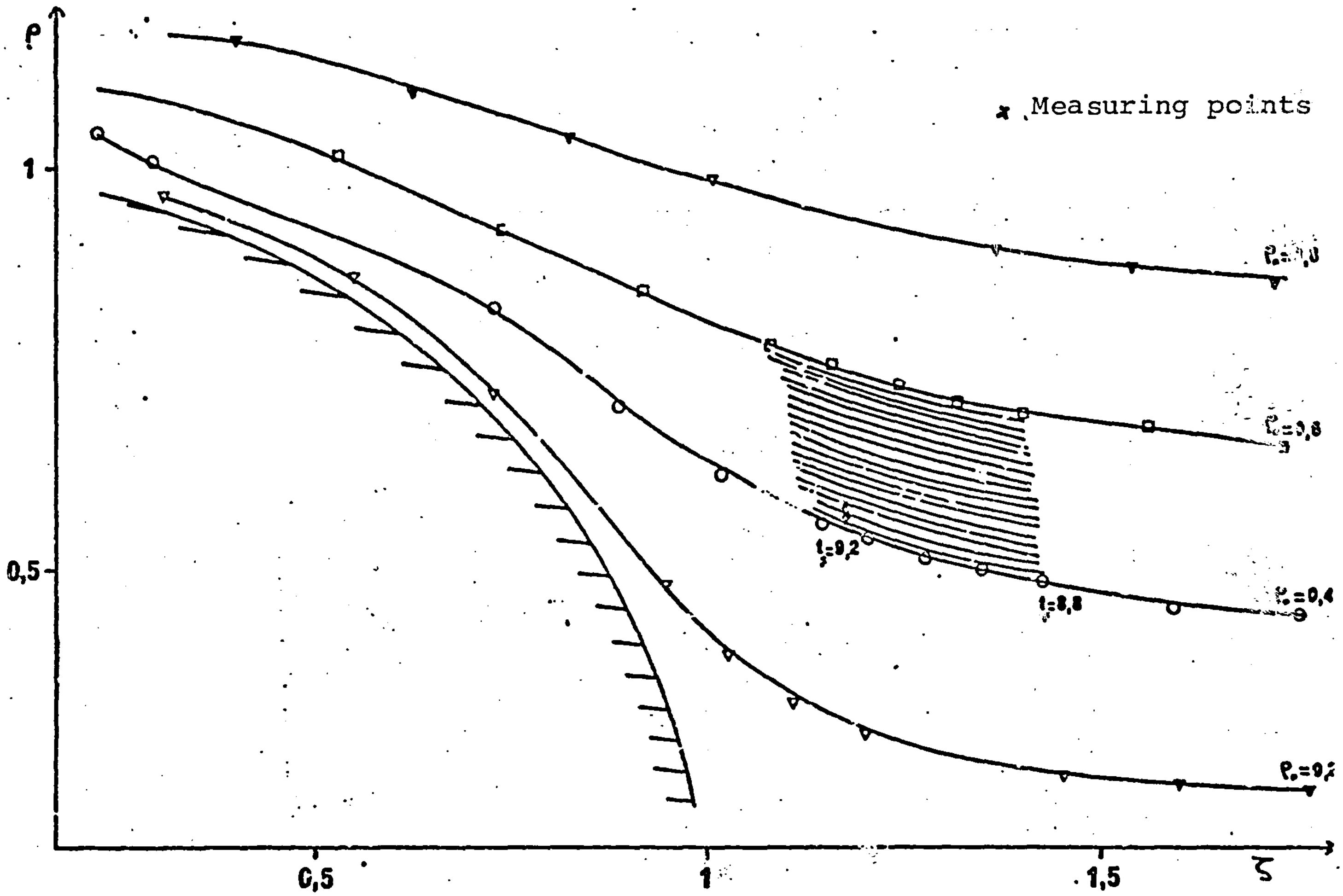


Fig. 2. Droplet trajectories for Sk = 0.01.

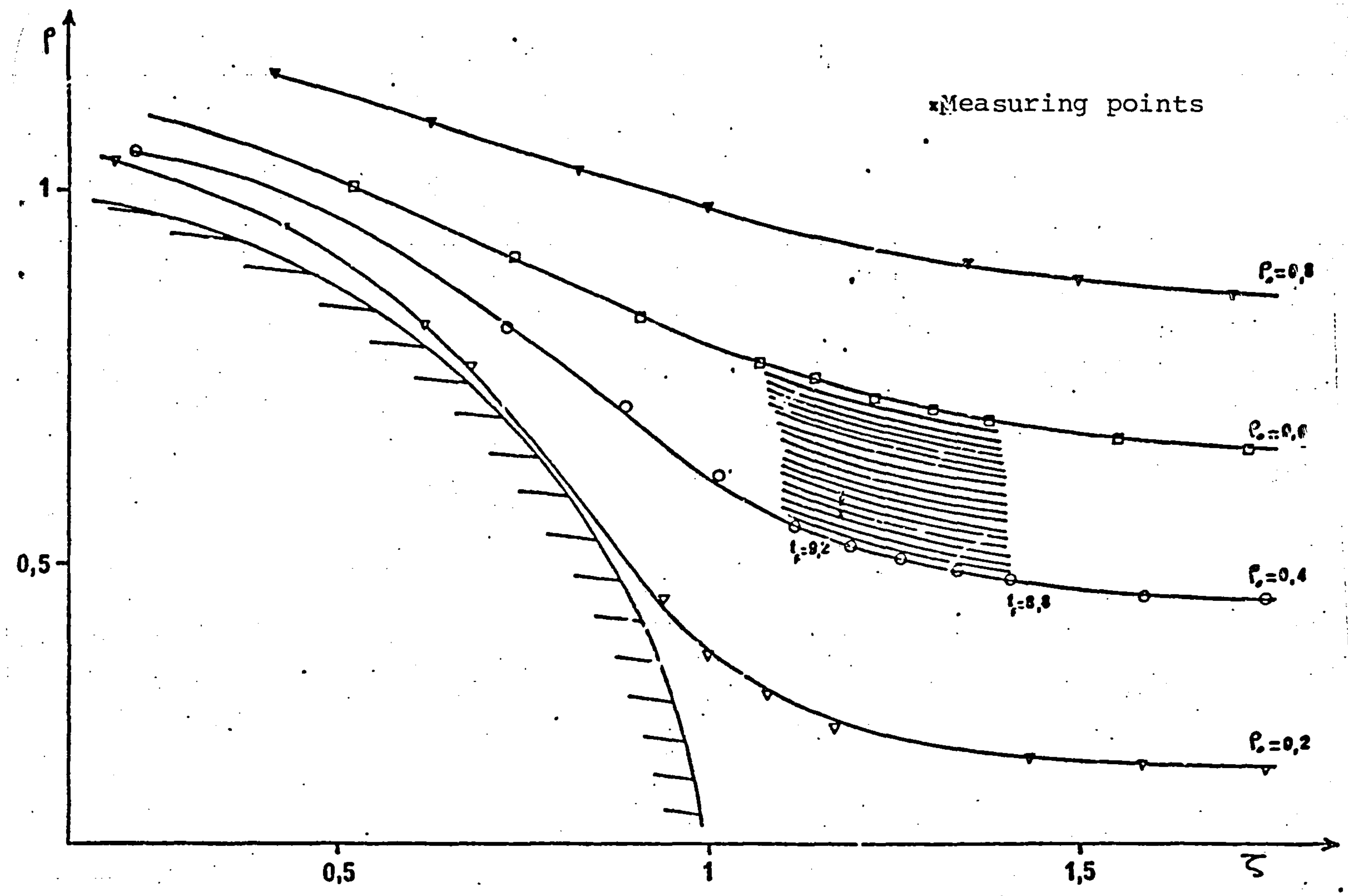


Fig. 3 - Droplet trajectories for Sk - 0.1.

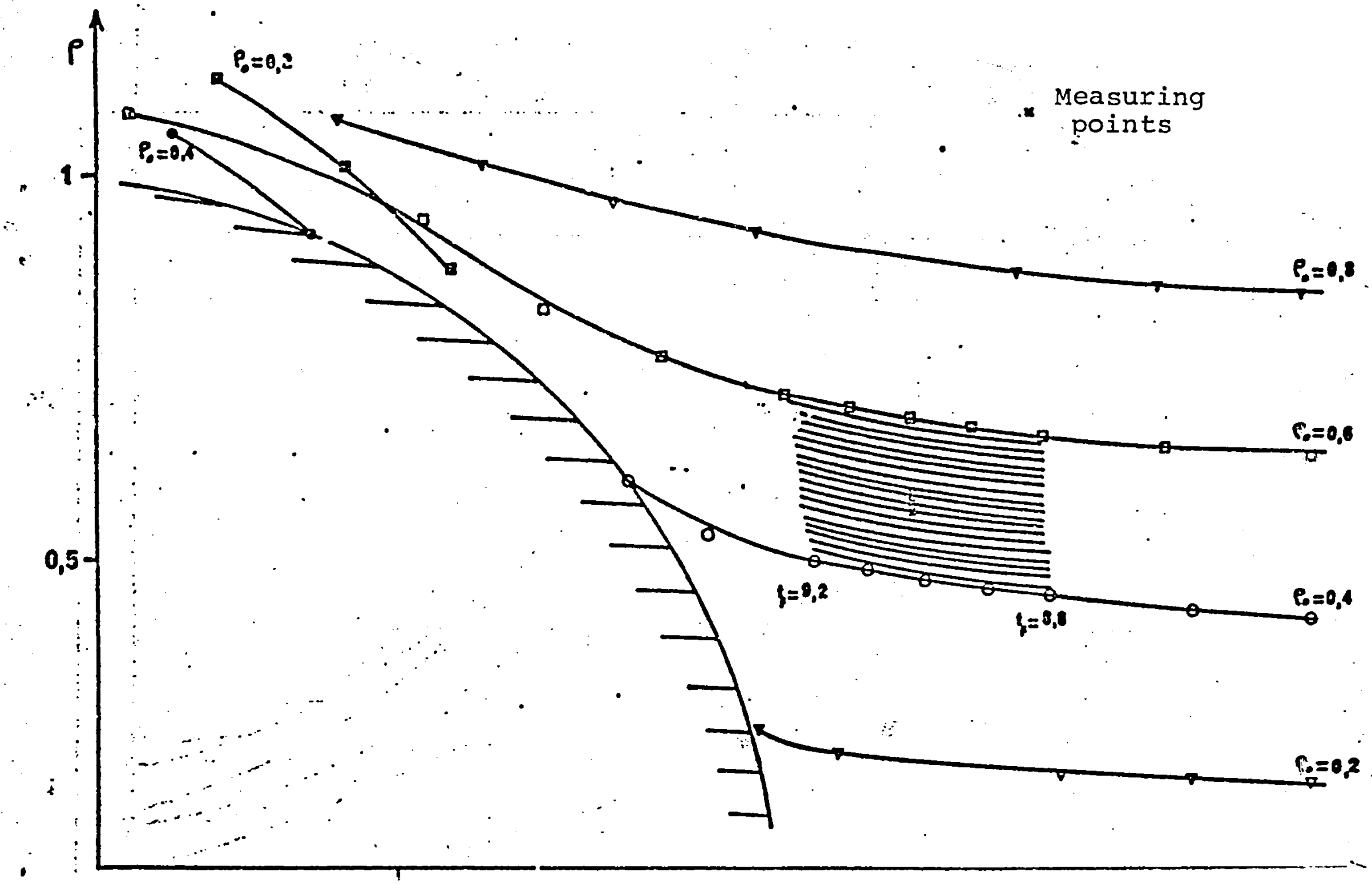


Fig. 4*

*Cut off on original text.

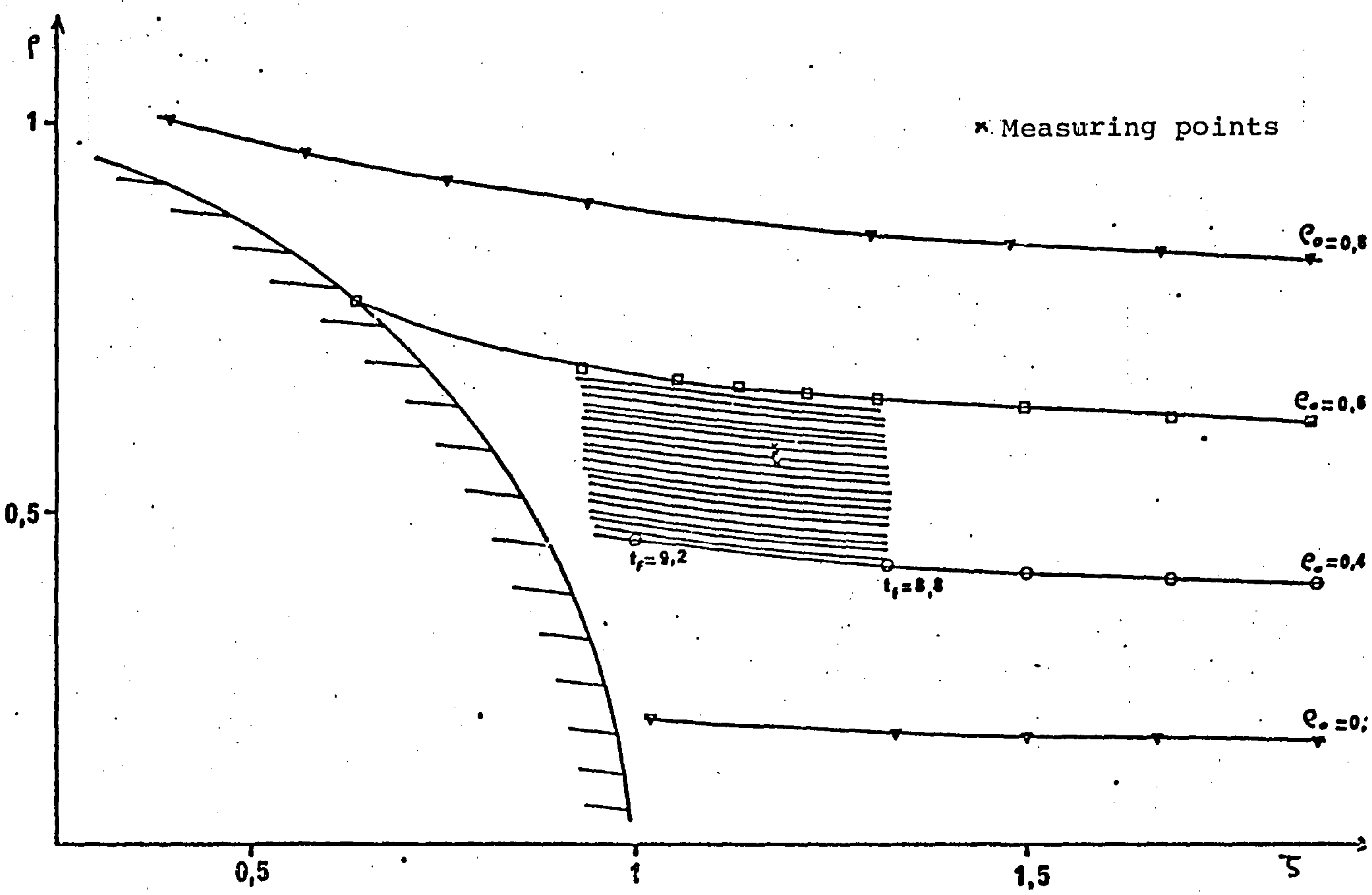
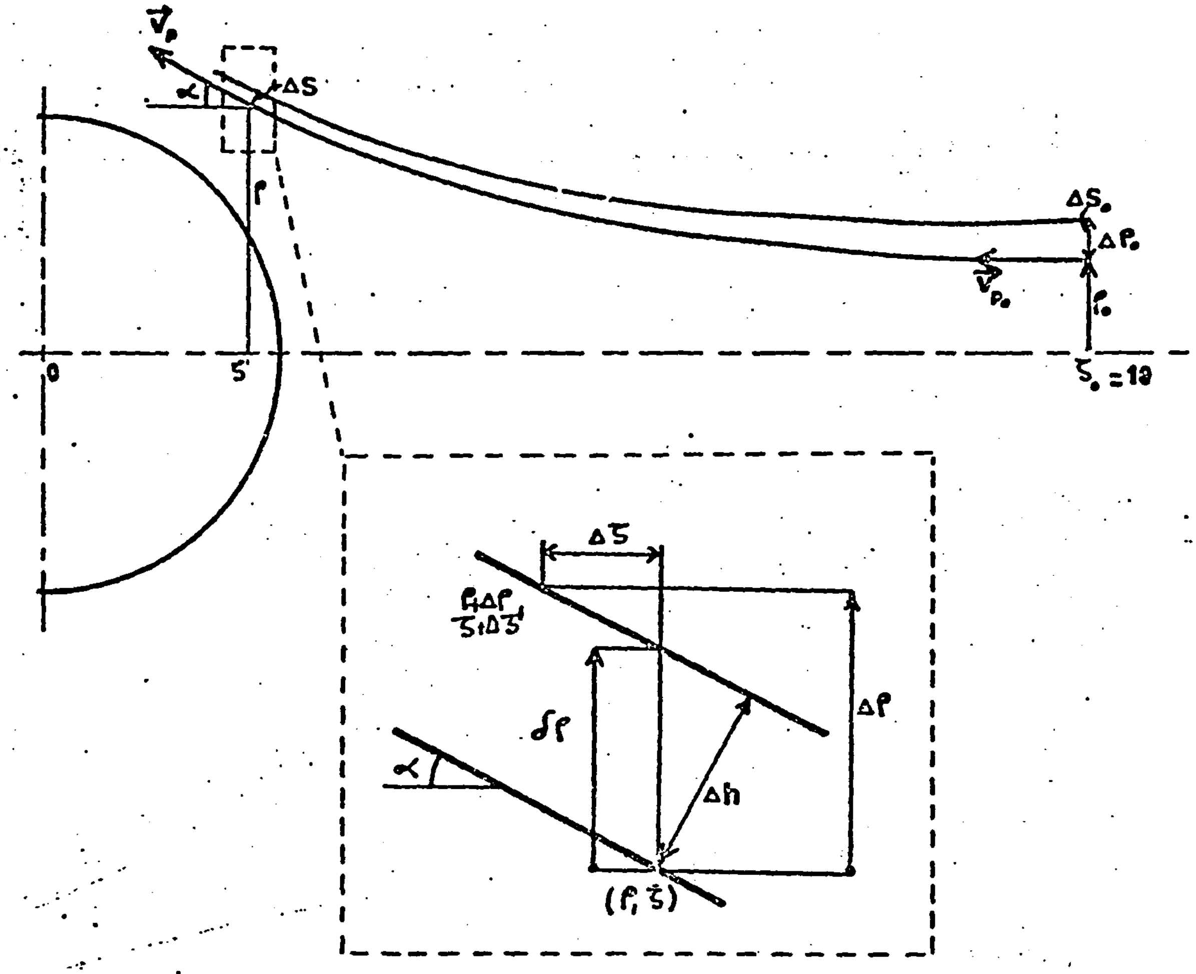


Fig.5. Droplet trjaectories for Sk = 1.



ΔS
ΔS₀
ΔP₀
0
5
ζ₀ = 10
Δζ
Δh
ΔP
δP

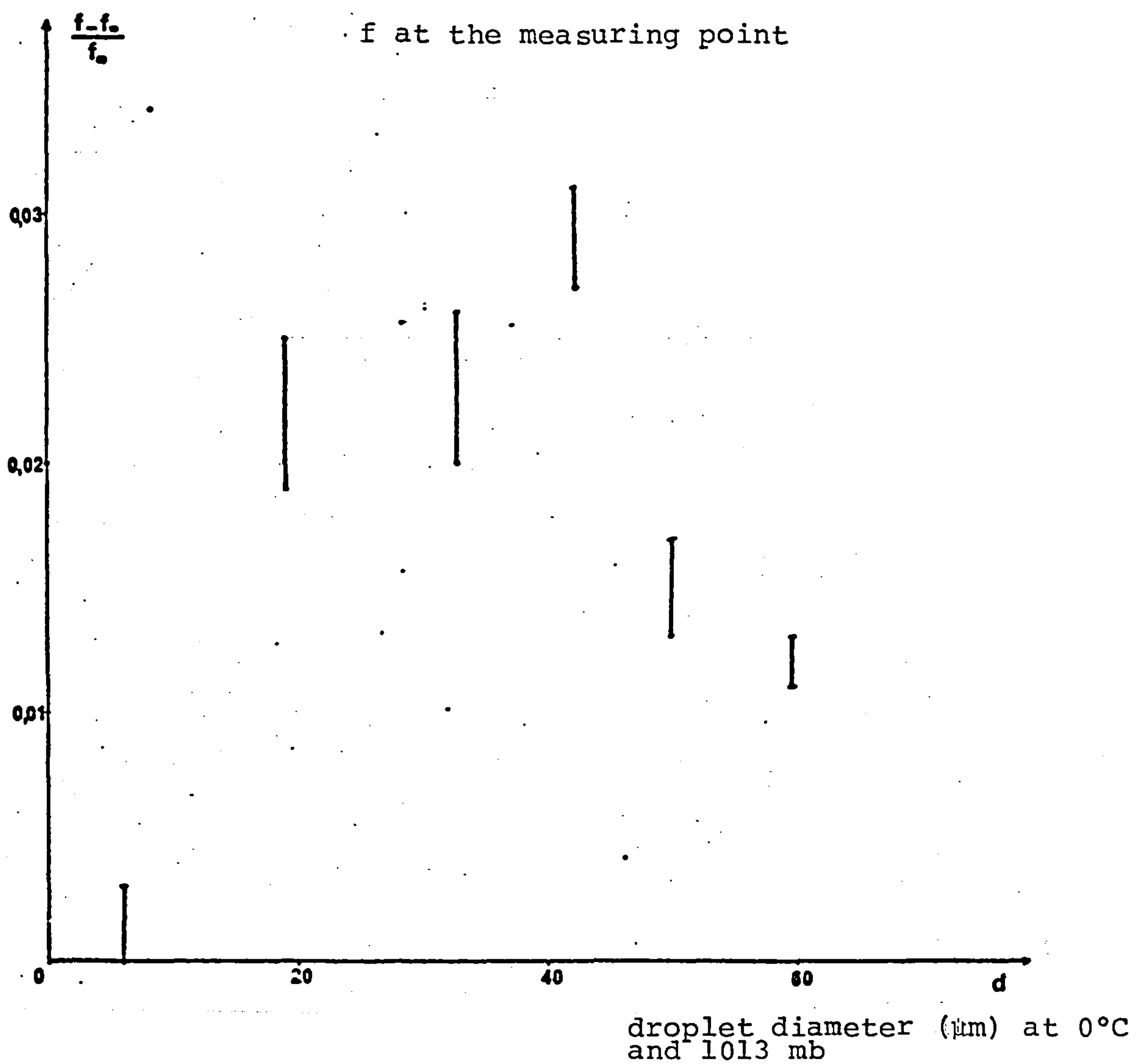


Fig. 7. Concentration values as a function of the droplets diameter.

Relative increase of concentration per unit of mass at the measuring point with respect to the concentration per unit of mass at infinity as a function of droplet diameter in normal conditions.

(The segments represent error ranges in the numerical calculation).

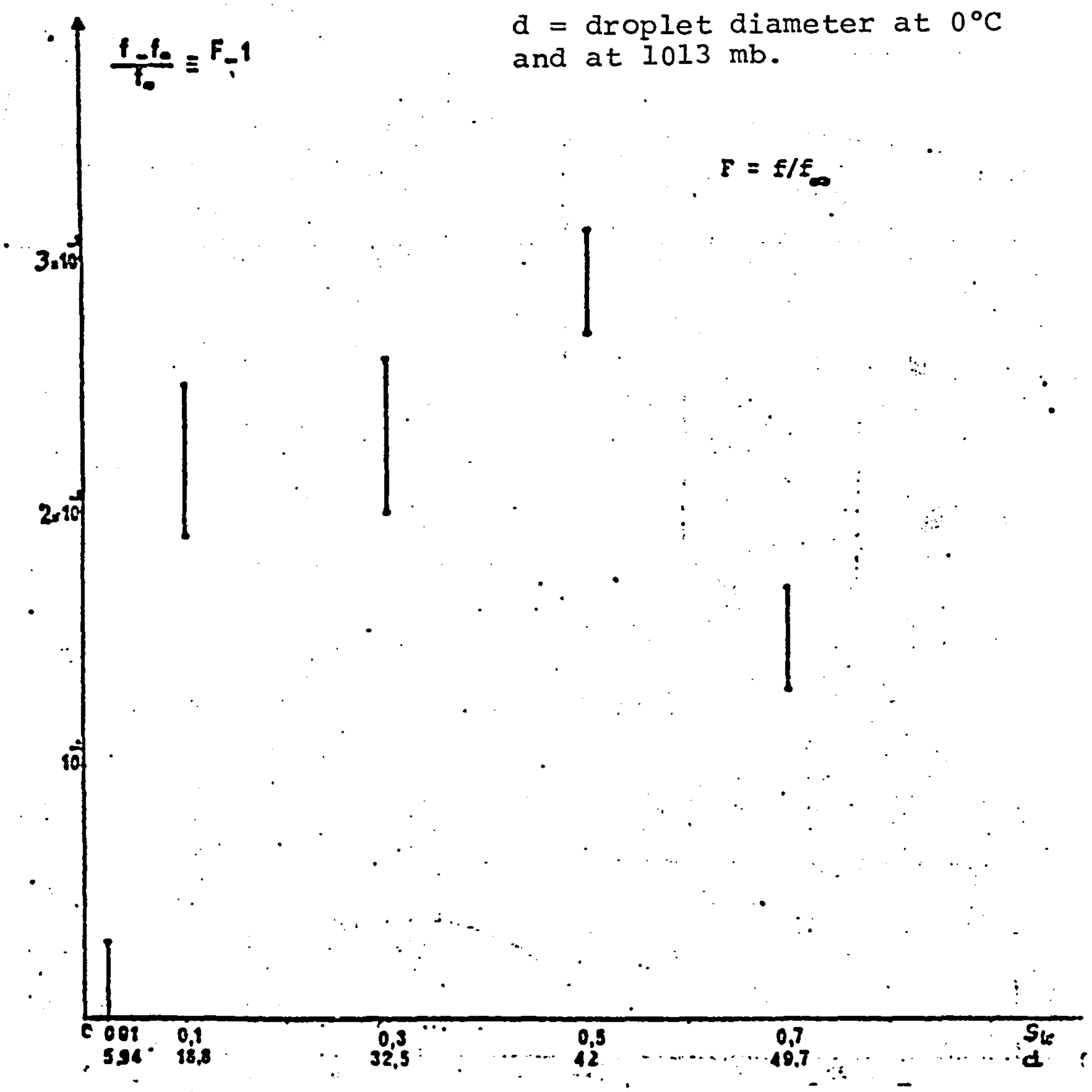


Fig. 8 - Concentration values as a function of Sk.

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